Effect of laser spot weld energy and duration on melting and absorption

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To facilitate pulse Nd–YAG laser spot weld development, it is common practice to adjust the pulse energy, duration, and focus spot size. An accurate understanding of the effect of these parameters on melting, weld appearance, and heat input is thus required. Calorimetric measurements of the net heat input to 304 stainless steel workpieces for laser spot welds have been completed. A pulse Nd–YAG laser was used with varying pulse energies from 1 to 5·5 J, and pulse durations of 2·2 and 7·0 ms. Measurements showed the absorption for spot welds produced using the pulsed Nd–YAG laser to vary from 38 to 67% and to be relatively insensitive to beam intensity. Analysis of the continuous point source equation for conduction heat flow in solids was used to predict the weld size for the pulse energy and duration measured in the experiment. Calculations of the weld pool volume from the weld metallography were used to determine the melting for each spot weld. Comparisons of the measured weld size with the three-dimensional model predicted size indicated that the observed weld pools are larger than is expected from the measured workpiece energy. Analysis of the experimental data and the theoretical model has revealed a substantial increase in melting for short duration pulses versus long duration pulses of the same energy. The benefit of laser spot welding parameter optimisation is hence indicated. STWJ/298

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INTRODUCTION

Pulse Nd–YAG lasers continue to replace the resistance spot welding process for joining of small electronic packages and other hardware assemblies. Laser spot welding has an important advantage for these applications because it can deliver a very precise amount of weld energy (0·1 to 50 J) in a very short time (0·1 to 20 ms), to a very small location. In addition, lasers do not contact the workpiece and can be accurately located on the weld joint through the use of focusing optics. The ubiquitous television gun is an example of an electronic assembly that requires up to 130 individual laser spot welds for completion. An important new application of laser spot welding has emerged in the fibre optic communications industry. Laser spot welding is now used for joining of wavelength dispersive multiplexing fibre optic connectors. Precise alignment of the fibre in the connector is obtained using active alignment laser spot welding machines.

Because of variability in light energy absorption by the metal surface, in the vapour plume, and in the laser supported vapour cavity, the actual fraction of laser pulse energy that is used to produce a given size weld is unknown.[1] To produce similar welds the pulse parameter levels selected often vary widely since absorption is not understood and the true effect of process parameters is not evident. Successful spot welds are certainly prevalent but they are achieved through trial and error, rather than informed consideration of parameter levels and their interactions.

Laser pulse energy, duration, and spot size are the key process parameters adjusted to achieve the desired laser spot weld dimensions, appearance, and heat input. Weld dimensions are the first requirement of laser spot welding since strength and alignment are often critical measures of quality. Weld appearance not only includes pool size but is also clearly affected by the occurrence of vaporisation and spatter. Owing to the presence of a luminous and audible vapour plume, users of the process are keenly aware of the onset of vaporisation, and the likelihood of spatter and drilling if laser beam intensity becomes excessive. Knowledgeable weld parameter selection is often intended to minimise vaporisation and spatter, which can result in significant weld pool defect formation, and contamination of adjacent package features. Parameter selection is also concerned with heat input, since the process is often chosen for temperature sensitive hardware that requires minimal heating and distortion. Despite the widespread use of laser spot welding, the effect of parameter selection on heat input is effectively unknown to most users, primarily because absorption is highly uncertain.

To increase the understanding of absorption and parameter selection in pulse Nd–YAG laser spot welding, calorimetric measurements of the net heat input to 304 stainless steel workpieces have been completed. Analysis of the new experimental data has revealed important effects of pulse energy, duration, and spot size on melting and heat input. The continuous point source equation for conduction heat flow in solids has been used to analyse the new data and appears to be useful for optimisation of the process.

EXPERIMENTAL PROCEDURE

Pulsed Nd–YAG laser spot welds were produced using a Raytheon SS501 pulse Nd–YAG laser with fixed optics beam delivery. The average power of the pulsing laser was measured using an Optical Engineering power probe, model P500Y. The laser beam was focused by a 150 mm focal length planoconvex lens with the focal plane located at the workpiece surface. A 4·9 mm intracavity aperture was installed on the laser. Laser spot size was measured using a Prometec UFF100 Laserscope with a 20 μm needle. The Laserscope was used to trigger the pulsed laser at 25 Hz to obtain spot size measurements corresponding to the three average power levels in Table 1. The Laserscope measurements were made using different pulse parameters but the same average power, since spot size for this solid state laser design is proportional to average laser power. Nominal pulse durations selected were 2·2 and 7·0 ms. No temporal pulse shaping was employed.

The continuously pulsing laser beam was rastered across plate samples to produce a series of non-overlapping spot welds by programmed translation of a computer numerical

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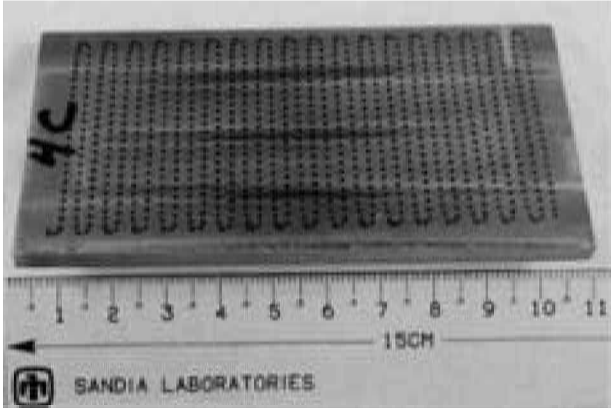


1 Representative 304 stainless steel plate sample with 900 individual Nd–YAG spot welds

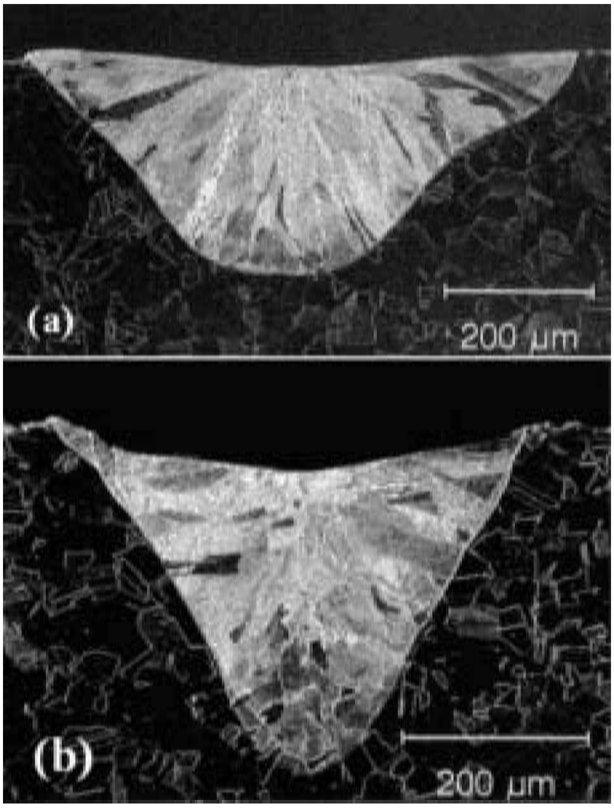
controlled table. Up to 1440 individual spot welds were made on each of 25 plates, which were welded and tested in the calorimeter. The plates had a wrought surface finish except for three, which had a milled surface finish. Each 304 stainless steel plate measured 55 ×104 ×6 mm. A representative plate with the milled finish and 900 spot welds is shown in Fig. 1. Supplementary argon shielding of the plate surface during welding was provided to protect the lens and reduce oxide formation.

The aggregate heat input to each plate was measured after welding using a Thermonetics Seebeck envelope calorimeter. The calorimeter walls were maintained at room temperature with a constant temperature bath. The calorimeter operates on the gradient layer principle[3] and produces a voltage output that is proportional to the flux through the walls during the time required for the weld sample to cool to room temperature. The calorimeter output was recorded using a digital oscilloscope then integrated to determine the total energy in joules absorbed by the workpiece for the number of spot welds made. Average absorption was determined by dividing the total energy by the number of spot welds on the plate. The average weld time was about 36 s. A rapid travel speed of 51 mm s^{-1} was required to assure that energy losses during this time were kept low. The movement of the plate during the laser pulse was estimated to be 0·36 mm for the 7·0 ms duration welds and 0·11 mm for the 2·2 ms duration welds. Despite the significant sample translation during the laser pulse, the weld pools appeared symmetrical with no discernible geometry effect despite the substantial movement difference between the two pulse durations.

Owing to the difficulty of sectioning the tiny spot welds only 15 of the 25 calorimetric plates were cross-sectioned to reveal the fusion zone. Examples of these welds are shown in Fig. 2. Longitudinal metallographic cross-section measurements through several collinear welds for each plate were averaged to determine weld pool width, depth, and volume. Pool volumes were calculated by dividing the metallographic cross-section from top to bottom into six rectangles, measuring the rectangular areas, and then summing the volumes of the six cylinders created by revolving the rectangles.

Table 1 Experimental conditions

Material	304 stainless steel
Pulse energy	1–5·5 J
Focusing lens	150 mm planoconvex
Focus spot diameter at surface	340, 420, 460 µm
Pulse duration	2·2, 7·0 ms
Peak power	176–785 W
Average power	40, 80, 110 W
Pulse frequency	15–40 Hz
Shielding gas	Argon
Travel speed	51 mm s^{-1}
Spot spacing	1·3–3·4 mm
Number of spot welds	540–1332 per plate
Thermal diffusivity	5·7 $mm^2 s^{-1}$
Thermal conductivity	34·1 W $m^{-1} K^{-1}$
Heat of vaporisation[2]	6330 kJ $kg^{-1} K^{-1}$
Enthalpy of melting	8·7 J mm^{-3}
Liquidus temperature	1727 K



a 2·6 J, 7·0 ms, spot radius 210 µm, absorption 0·55; *b* 1·2 J, 2·2 ms, spot radius 170 µm, absorption 0·61

2 Representative pulse Nd–YAG laser spot weld cross-sections in 304 stainless steel (optical)

Predicted values of weld pool size and depth for the transient conduction problem were calculated with the aid of a desktop computer and the continuous point source solution for conduction heat flow in a semi-infinite body given by Bahun and Engquist,[4] which is as follows

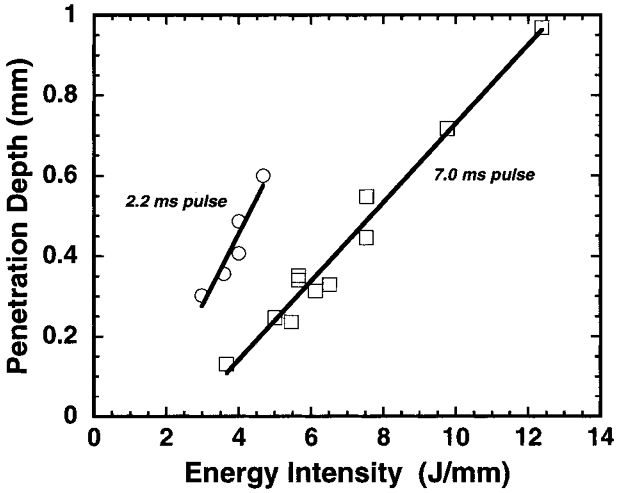
$$T(r,t)=\frac{q}{2\pi kr}\,\mathrm{erfc}\left[\frac{r}{(4\alpha t)^{1/2}}\right] \qquad (1)$$

where T is temperature at a given radius r from the source and time t, q is source input power, k is thermal conductivity, and α is thermal diffusivity.

If it is assumed that the temperature produced in the part is the combined sum of two sources, one heating during the laser pulse and the other cooling after the pulse, the principle of superposition as described by Rykalin[5] can be used to find a solution for the temperature at any time after the pulse is terminated, according to

$$\mathrm{f}(r,t)=T(r,t)-\frac{q}{2\pi kr}\left(\mathrm{erfc}\left[\frac{r}{(4\alpha t)^{1/2}}\right]\right.$$
$$\left.-\mathrm{erfc}\left\{\frac{r}{[4\pi(t-t_{\mathrm{p}})]^{1/2}}\right\}\right) \qquad (2)$$

where t_{p} is the pulse duration time and it is assumed that $t>t_{\mathrm{p}}$.



3 **Effect of laser energy intensity and pulse duration on 304 stainless steel spot weld penetration depth**

Using a numerical method, equation (2) is solved for the (r,t) combination that gives the largest extent $(r=r_{max})$ of the melt zone, for which $T(r,t)=T_{liq}$: the material properties used are given in Table 1. The *fmin* function in Matlab was used for a one-dimensional search in the following manner.

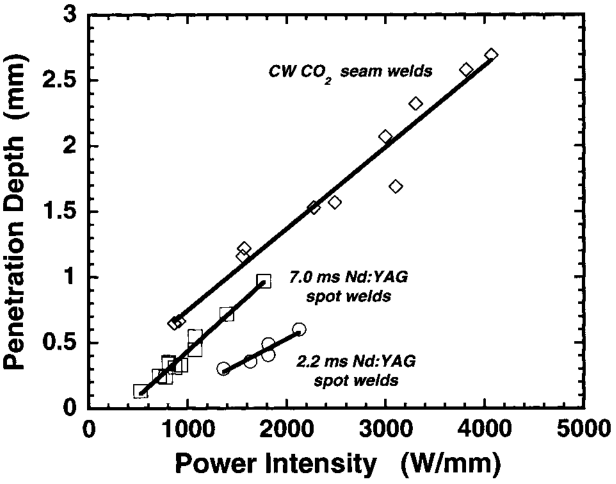
1. Specify a time interval $t_{min} \leqslant t \leqslant t_{max}$ in which r_{max} occurs.
2. Solve for $r_{max}(t)$ using a one-dimensional search over the interval in step 1 as follows:
   - (i) *fmin* chooses a $t=t^*$ from the time interval in step 1
   - (ii) *fmin* function evaluation solves $f(r,t^*)=0$ for r, given t^*, and $T(r,t^*)=T_{liq}$ using combination of bisection and Newton's methods over a guessed range of r values
   - (iii) *fmin* iterates on t^* until $r=r_{max}$.

Since equation (2) is for a point source, it does not take into account the laser spot diameter. The continuous point source model also assumes that the material thermal properties are independent of temperature and that there is no thermal effect due to latent heat of fusion or material vaporisation.

RESULTS AND DISCUSSION

Fusion zone characteristics

The strong dependence of weld penetration on energy intensity is shown in Fig. 3. Energy intensity is defined as the laser output pulse energy divided by the measured spot diameter at the plate surface. Figure 3 provides an unequivocal illustration of the effect of laser energy intensity and pulse duration on laser spot weld penetration depth. As was found in Ref. 6, normalising the effect of power by the focus spot size results in a stronger correlation than the use of the more typical focus spot area. For predicting laser weld responses such as penetration depth and pool volume, focal spot area does not appear significant. The reason for this is uncertain but it may indicate that laser beam weld interaction is more accurately represented by a point or line source than a surface source. It is important to note in Fig. 3 that for the same energy and spot size the short duration pulses result in greater penetration than the long duration pulses. In other words, if equivalent laser energy is transferred to the workpiece by pulses of different durations, increased penetration will occur for the shorter duration pulse. When weld penetration is plotted against power intensity (peak power divided by spot diameter) as shown in Fig. 4, the relative importance of pulse duration is reversed and the long duration pulses result in deeper penetration for the same intensity. This result is somewhat more intuitive, in that it would be expected that for the same power and spot size, a longer pulse should achieve greater melting and deeper penetration. A conduction model will be presented below that is also useful in explaining the non-intuitive pulse duration relationship found in Fig. 3.



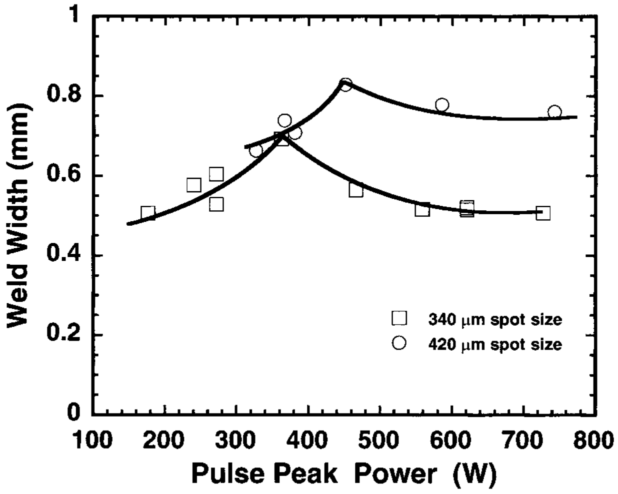
4 **Effect of both continuous wave (CW) CO_2 and pulse Nd–YAG power intensity on 304 stainless steel penetration depth: CO_2 seam welds produced at 5·1 mm s^{-1} (data from Ref. 6)**

Also shown in Fig. 4 is the dependence of weld penetration on power intensity for continuous wave CO_2 laser welds made on 304 stainless steel in Ref. 6. It can be seen that significantly deeper laser welds can be produced using the continuous power laser than were made in the present study using the pulsed laser. Deep penetration welds were not feasible using the pulsed Nd–YAG laser at high beam intensities owing to the onset of significant spatter, and no welds deeper than 1 mm were produced. The classic keyhole like vapour cavity that is common in continuous wave seam welding is to be avoided in pulse laser spot welding since a penetrating impulse can lead to spatter and its consequences. Also note in Fig. 4 that at low laser power intensities, the pulse Nd – YAG weld penetration is comparable in magnitude to the continuous wave CO_2 penetration. The relative ranking of the two lasers depends primarily on the pulse duration examined.

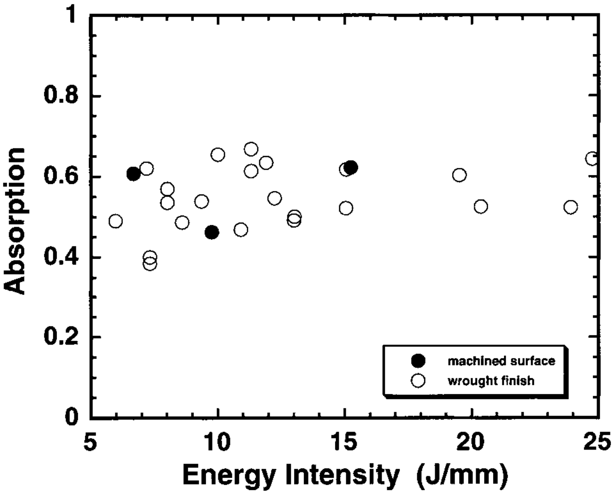
Weld pool width is an important parameter for laser spot welding, since for many applications, bridging joint gaps and hermeticity are more important goals than reaching a specific penetration depth or mechanical strength. The effect of laser spot size on weld pool depth can be seen in Fig. 5. It is apparent that a larger spot size can yield a wider pool width for spot welds using the same peak power. It is also worth noting that actual weld pool width ranges from 50 to 100% greater than the focused laser spot diameter. This dependence of weld pool width on focused beam diameter is similar to behaviour observed for pulse Nd–YAG seam welding of pacemaker batteries. Changes in focus spot size were obtained in that work by varying lens focal length.[7] The separate measurements of pulsed laser focus spot diameter at specific powers in the present work were obtained for a single lens. Whereas it appears that pulse power has little effect on pool diameter, the focus spot size does significantly affect the resulting pool geometry.

Energy absorption

The average absorption (fraction of laser pulse energy absorbed by the plate and measured using the calorimeter) is shown for each of the 25 plates in Fig. 6. The solid points in Fig. 6 indicate welds that were made on plates that had a machined surface finish. No significant effect of surface



5 **Effect of Nd–YAG laser focus spot diameter on 304 stainless steel spot weld pool width**



6 **Pulse Nd–YAG spot weld absorption on 304 stainless steel for various levels of pulse energy, duration, and spot size**

finish on absorption can be discerned for these three specially prepared plates. The measured absorption for all welds varies from 0·38 to 0·67. Although some data scatter is apparent in the figure, these values are surprisingly constant despite significant changes in energy, intensity, and pulse duration. It is noteworthy that the absorption values are almost all above 0·40 and yet no absorption values are higher than 0·70. These results are similar in magnitude to pulsed Nd–YAG laser spot weld values determined by Cremers *et al.*[8] from volumetric expansion of 316 stainless steel samples: in those experiments absorption values were found to vary from 0·21 to 0·62.

The magnitude of the maximum absorption measurements in Fig. 6 is lower than expected. Previous CO_2 laser continuous wave seam welds on 304 stainless steel have shown that energy transfer efficiencies as high as 92% are typical.[6] This difference in laser type performance is surprising since it is opposite to the expectation. Absorption is thought to be greater for Nd–YAG lasers than for CO_2 lasers, since room temperature absorption values for the 1·06 μm wavelength Nd–YAG laser are known to be higher than for the longer 10·6 μm wavelength CO_2 laser.[9] It is therefore expected that the Nd–YAG laser will achieve a similarly high energy absorption when welding.

It is entirely possible that the low absorption values measured in the present work are legitimate and are simply the result of a non-ideal fusion zone geometry. Fresnel absorption occurs when the laser beam reflects and absorbs multiple times inside a deep keyhole like vapour cavity;[10,11] this is the mechanism considered responsible for the absorption values greater than 0·90 obtained in the previous CO_2 seam weld study. Those calorimetric measurements showed a strong correlation of absorption with power intensity. No strong correlation of absorption with power intensity is shown in Fig. 6, nor was any correlation found for any other independent parameters. The present spot weld data show little dependence of absorption on weld penetration or fusion zone geometry; the relatively shallow penetration depths in the present experiment (*see* Fig. 4) may be responsible. Multiple reflections inside the vapour cavity may be insufficient to obtain the same high absorption as was observed for CO_2 seam welding.

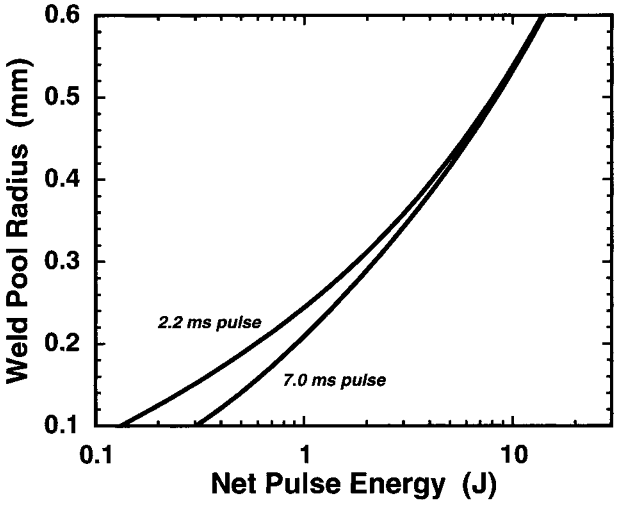
It is also important to note that to obtain coupling in continuous wave CO_2 laser welding, the focused laser beam is typically much smaller than for Nd–YAG spot welding. The focused laser spot diameter for the CO_2 welds in Fig. 4 is 118 μm, whereas the spot diameter for the Nd–YAG spot welds in Fig. 4 varies from 340 to 460 μm. A similarly large spot size in the continuous wave CO_2 process does not produce any significant melting. The two processes behave quite differently in this characteristic. The small CO_2 spot size should create a narrow vapour cavity, promote a greater number of beam reflections, and thereby enhance laser beam absorption.

Energy losses

Caution is necessary in considering the results of Fig. 6. The absorption values shown could indeed be artificially low if radiation, convection, vaporisation, or spatter are significant sources of energy loss from the weld plate. The calorimetric technique used in the present experiment has no means of capturing these potential losses, since they might occur before the calorimeter is closed. Previous experiments have established that the potential for significant radiative loss is low owing to the small surface area of the weld.[12] A conservative estimate of the convective loss due to argon gas shielding based on Newton's law of cooling reveals a convective loss of less than 10 mJ in total during and between pulses. For most spot welds this value is less than 1% of the net pulse energy. Losses due to spatter are thought to be low since weld parameters were selected by observation to minimise it. Nonetheless spatter was not measured and a small amount cannot be ruled out, especially for the high energy intensity welds.

Vaporisation losses from the plate are potentially more significant than the other losses. Mass loss measurements by Akau *et al.*[13] for Nd–YAG spot welds on 304 stainless steel showed that 165 μg total mass per pulse was lost after welding. If it is assumed that all of the 165 μg mass is vaporised, and the value for latent heat of vaporisation of stainless steel is used,[2] then 1·0 J of energy, a very significant quantity, could be lost from the sample as a result of vaporisation. Mass loss measurements for continuous wave CO_2 laser welding on stainless steel show a much smaller effect. Using the maximum vaporisation rate measured by Khan and DebRoy[14] for 202 stainless steel, only a 12 μg mass loss would be expected during a 20 ms melting time.

The disparate results in vaporisation magnitude are intriguing. Perhaps mass loss is indeed lower for CO_2 laser seam welds than for pulse Nd–YAG spot welds. Laser beam interaction with the vapour plume is known to be specific to the laser wavelength, with CO_2 lasers developing a much more intense high temperature plasma than Nd–YAG lasers.[15] Laser energy absorption or transmission in the vapour plume may affect the pressure gradient at the weld pool surface and thereby control the vaporisation rate.



7 **Point source model prediction for dependence of spot weld depth (hemisphere radius) on pulse duration and energy for 304 stainless steel**

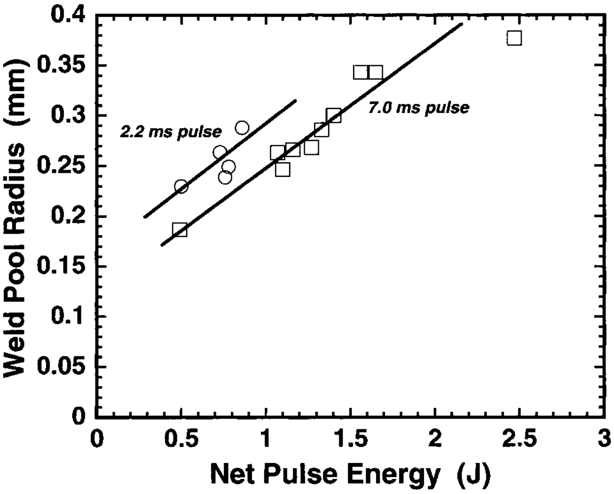
Is should also be considered that the measurement by Akau *et al.*[13] is for a 24 J pulse, which is more than 4 times the maximum energy level used in the present experiment. Notwithstanding the discrepancy in evaporation magnitude, it seems certain that a significant amount of energy loss through vaporisation is possible for Nd–YAG spot welds and may contribute to an undermeasurement of absorption. Only additional experimentation can quantify this effect.

Conduction model predictions

The dependence of spot weld pool size on pulse energy and duration can be predicted from the continuous point source solution to the conduction heat flow equation given in equation (2). Since the point source solution assumes a hemispherical weld pool, only low aspect ratio welds should be compared with this model. The model is justified for the present work since no welds have greater than 1 mm penetration depth (*see* Fig. 2). The dependence of weld pool depth (hemisphere radius) on pulse energy and duration for 304 stainless steel is shown in Fig. 7. It is interesting to find that the advantage of short duration pulses in producing melting that was observed in Fig. 3 is indeed predicted by the conduction theory model. It can be seen in Fig. 7 that at low pulse energies, the advantage of a short duration pulse in producing melting can be substantial. For example, Fig. 7 indicates that a 0·5 J pulse for 2·2 ms will produce a weld pool volume that is ×2·3 larger than that for a 0·5 J pulse for 7·0 ms. The predicted difference is remarkable and can be validated by the present experimental results.

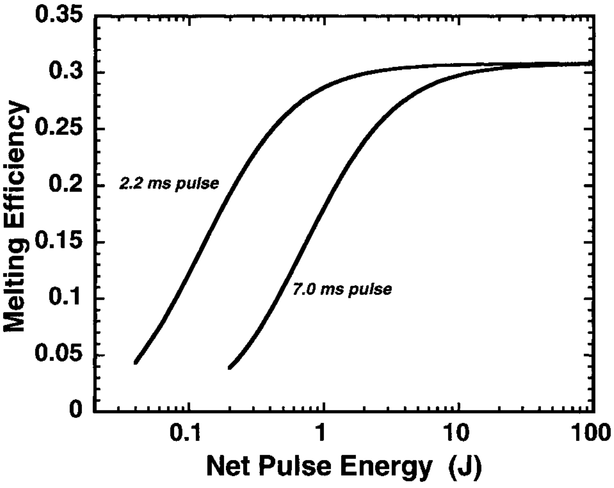
Although the data in Fig. 3 support the model predictions, Fig. 3 is not directly comparable to the model in Fig. 7, since the depth in Fig. 7 refers to a hemispherical pool and the energy represents pulse energy actually absorbed by the workpiece. Since the penetration depth shown in Fig. 3 is not a true indicator of melting, weld pool volume measurements were made on the metallographic sections to compare the experimental results with the model. In Fig. 8, the calculated pool radius corresponding to actual measured pool volumes is compared with the actual measured net pulse energy. It can be seen that the strong benefit of short duration pulses first shown in Fig. 3 can also be seen when weld pool volumes and actual net energy are compared.

The greater melting observed for the short duration pulse in Figs. 7 and 8 is due to an increase in the melting efficiency. Spot weld melting efficiency is the ratio of the energy necessary to just melt the fusion zone to the net energy absorbed by the workpiece. Once again, the significant advantage of short duration versus long duration pulses is obvious in Fig. 9, where the dependence of weld melting efficiency on pulse energy and duration for 304 stainless steel is shown. It is apparent that the advantage of the short duration pulse decreases as pulse energy increases and is eliminated entirely when the pulse energy is above approximately 30 J. It is important to remember that these results are for 304 stainless steel and that the curves in Figs. 7 and 8 will undoubtedly shift for other materials. The theoretical maximum melting efficiency of 0·31 in Fig. 9 predicted by the model for both pulse durations will however be the same for any material or pulse duration since it is determined by the heat flow geometry. Since laser welding is often selected to reduce heat input, the selection of laser spot weld pulse parameters should be based on careful understanding of the results presented in Figs. 7 and 9. Evidently an optimisation of pulse parameters can be used to yield the lowest heat input and the smallest heat affected zone.

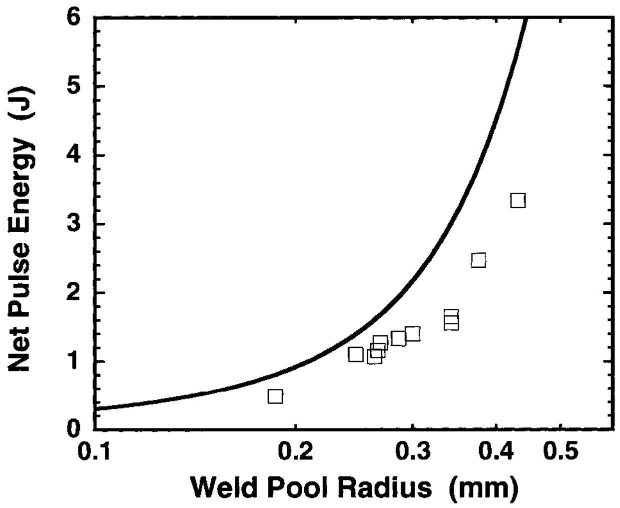


8 **Effect of pulse duration and net pulse energy on measured volumetric pool radius for 304 stainless steel**

The point source model is also useful in comparing the actual weld size with the size predicted for the net energy measured. The calculated pool radii for the measured pool



9 **Point source model prediction for dependence of melting efficiency on net pulse energy and pulse duration for 304 stainless steel spot welds**



10 Comparison of measured volumetric pool radius with point source model prediction (solid line) for 7·0 ms spot welds on 304 stainless steel

volumes are compared with the model predicted radii in Fig. 10 for 7 ms pulses on 304 stainless steel. Since for each pool radius the measured pulse energy is substantially below the energy predicted by the model to melt that pool, it can be inferred that the pulse energy has been undermeasured, and that absorption is lower than that suggested by the pool volume. As discussed above, significant vaporisation is the most likely cause of an artificially low absorption. If it is assumed for each spot weld that the difference in energy between the model prediction and calorimeter measurement is due entirely to energy lost by evaporation, the mass loss for each weld can be estimated. Figure 11 shows the estimated mass loss due to vaporisation per pulse for both the 2·2 and 7·0 ms spot welds. The results indicate that for the same power intensity, vapour loss is greater for a longer pulse duration. The mass losses vary between 28 and 350 μg and are comparable to the 165 μg mass loss measured by Akau *et al.*[13] If some spatter occurred for the high power intensity spot welds, the estimated vapour loss for those welds could be significantly less, since mass lost to spatter would contain less energy than mass lost to vaporisation. If the vapour loss is actually as significant as shown in Fig. 11, then an additional criterion for optimisation of pulse parameters may be suggested, namely, that optimisation should be used not only to yield the lowest heat input, but also to reduce weld pool vaporisation.

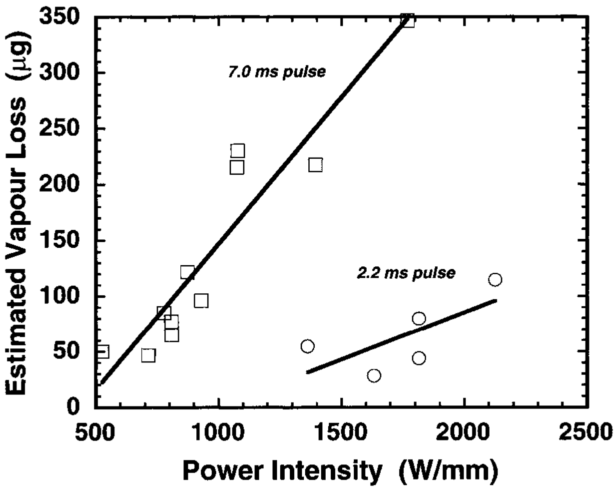
CONCLUSIONS

1. Calorimetric measurements of energy absorption for pulsed Nd–YAG laser spot welds on 304 stainless steel have been found to vary from 0·38 to 0·67 and to show no correlation with beam intensity or weld penetration.

2. Short duration 2·2 ms laser pulses resulted in greater weld penetration depth and melted volume than long duration 7·0 ms pulses of the same net energy.

3. Solutions to the continuous point source equation for conduction heat flow show a correlation with the increased melting observed for short duration pulses, and indicate that a substantial reduction in heat input can be obtained through optimisation of pulse energy and duration.

4. Predictions of weld pool volume using the continuous point source equation indicate that observed weld pools are larger than the calorimetrically measured energy should produce. An undermeasurement of pulse energy is a possible cause for this discrepancy between theory and experiment.



11 Estimated vaporised mass based on difference between predicted and measured weld pool volumes

5. Without independent measurements of spot weld mass loss it is impossible to be certain whether vaporisation is responsible for the low absorption values measured, or this low absorption is due to shallow penetration and a weld pool geometry that is not favourable to Fresnel absorption.

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